Introduction to RADAR systems

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chapter

5

Detection of Signals in Noise

5.1 INTRODUCTION

A radar *detects the presence* of an echo signal reflected from a target and *extracts information* about the target (such as its location). One without the other has little meaning. The detection of radar signals in noise was discussed in Chap. 2, detection of moving targets in clutter was the subject of Chap. 3, and detection of stationary targets in clutter is discussed in Chap. 7. In the current chapter, additional aspects of the detection of radar signals in noise will be presented, chiefly the matched filter and related topics. The extraction of information from a target echo signal is the subject of the following chapter, Chap. 6.

Methods for the detection of desired signals and the rejection of undesired noise, clutter, and interference in radar are called *signal processing*. The matched filter, described next, is an important example of a radar signal processor.

5.2 MATCHED-FILTER RECEIVER[1,2]

Under certain conditions, usually met in practice, maximizing the output peak-signal-to-noise (power) ratio of a radar receiver maximizes the detectability of a target. A linear network that does this is called a *matched filter*. Thus a matched filter, or a close approximation to it, is the basis for the design of almost all radar receivers.

Matched Filter Frequency Response Function The matched filter that maximizes the output peak-signal-to-mean-noise ratio when the input noise spectral density is uniform (white noise) has a frequency response function[1]

$$H(f) = G_a S^*(f) \exp(-j2\pi f t_m) \tag{5.1}$$

where G_a is a constant, t_m is the time at which the output of the matched filter is a maximum (generally equal to the duration of the signal), and $S^*(f)$ is the complex conjugate of the spectrum of the (received) input signal $s(t)$, found from the Fourier transform of the received signal $s(t)$ such that

$$S(f) = \int_{-\infty}^{\infty} s(t) \exp(-j2\pi f t)\, dt$$

(The matched filter that maximizes the output signal-to-noise ratio should not be confused with the circuit-theory concept of impedance matching, which maximizes the power transfer between two networks.)

The received signal spectrum can be written as $S(f) = |S(f)| \exp[-j\phi_s(f)]$, where $|S(f)|$ is the amplitude spectrum and $\phi_s(f)$ is the phase spectrum. Similarly, the matched filter frequency-response function can be expressed in terms of an amplitude and phase as $H(f) = |H(f)| \exp[-j\phi_m(f)]$.

Letting the constant G_a equal unity, we can use these relations to write Eq. (5.1) as

$$|H(f)| \exp[-j\phi_m(f)] = |S(f)| \exp\{j[\phi_s(f) - 2\pi f t_m]\} \tag{5.2}$$

Equating the amplitudes and phases in the above gives

$$|H(f)| = |S(f)| \tag{5.3}$$

$$\phi_m(f) = -\phi_s(f) + 2\pi f t_m \tag{5.4}$$

It is seen that the magnitude of the matched-filter frequency-response function is the same as the amplitude spectrum of the input signal, and the phase of the matched-filter frequency response is the *negative* of the phase spectrum of the signal plus a phase shift proportional to frequency. The effect of the negative sign before $\phi_s(f)$ is to cancel the phase components of the received signal so that all frequency components at the output of the filter are of the same phase and add coherently to maximize the signal.

Matched Filter Impulse Response The matched filter may also be described by its *impulse response* $h(t)$, which is the inverse Fourier transform of the frequency response function $H(f)$ of Eq. (5.1), or

$$h(t) = \int_{-\infty}^{\infty} H(f) \exp(j2\pi f t)\, df = G_a \int_{-\infty}^{\infty} S^*(f) \exp[-j2\pi f(t_m - t)]\, df \tag{5.5}$$

Since $S^*(f) = S(-f)$, Eq. (5.5) becomes

$$h(t) = G_a \int_{-\infty}^{\infty} S(f) \exp[j2\pi f(t_m - t)]\, df = G_a s(t_m - t) \tag{5.6}$$

The expression on the far right comes from recognizing that the integral is an inverse Fourier transform. Equation (5.6) indicates that the impulse response of a matched filter

is the time inverse of the received signal. It is the received signal reversed in time, starting from the fixed time t_m. Figure 5.1 shows an example of the impulse response $h(t)$ of the filter matched to a signal $s(t)$.

The impulse response of a filter, if it is to be realizable, must not have any output before the input signal is applied. Therefore, we must have $(t_m - t) > 0$, or $t < t_m$. This is equivalent to the condition on the frequency response function that there be a phase $\exp(-j2\pi f t_m)$, which implies a time delay of t_m. For convenience, the impulse response is often written simply as $s(-t)$ and the frequency response function as $S^*(f)$, with the realizability conditions understood.

Receiver Bandwidth The matched filter is implemented in the IF stage of a superheterodyne receiver since the bandwidth of a superheterodyne receiver is essentially that of the IF. (The bandwidths of the RF and the mixer stages are usually large compared to that of the IF.) Thus the maximum signal-to-noise ratio occurs at the output of the IF. The second detector and the video portion of the receiver have negligible effect on the output signal-to-noise ratio if the video bandwidth is greater than one half the IF bandwidth.

Derivation of the Matched-Filter Frequency Response The frequency response function of the matched filter can be derived using the calculus of variations[1] or the Schwartz inequality.[3] In this section, the Schwartz inequality is used.

We wish to show that the frequency-response function of the linear, time-invariant filter that maximizes the output peak-signal-to-mean-noise ratio is

$$H(f) = G_a S^*(f) \exp(-j2\pi f t_m)$$

when the input noise is stationary and white (uniform spectral density). The ratio to be maximized is

$$R_f = \frac{|s_0(t)|^2_{\max}}{N} \tag{5.7}$$

where $|s_0(t)|_{\max}$ is the maximum value of the output signal voltage and N is the mean noise power at the receiver output. (The ratio R_f is not quite the same as the

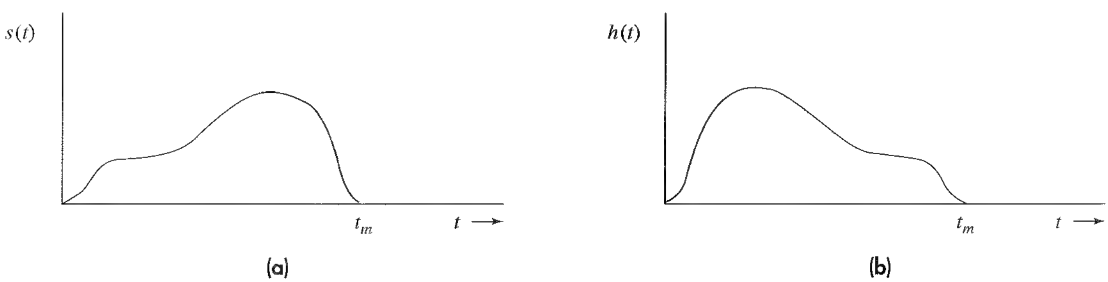


Figure 5.1 (a) Example of a received waveform $s(t)$; (b) impulse response $h(t)$ of the matched filter for the input signal $s(t)$ of (a).

signal-to-noise ratio considered previously in the radar range equation of Chap. 2. The peak power here is the peak *instantaneous* power, whereas the peak power in the discussion of the radar equation in Chap. 2 was the average value of the power over the *duration* of a pulse of sinewave. The ratio R_f is *twice* the average signal-to-noise ratio when the input signal $s(t)$ is a rectangular sinewave pulse.) The magnitude of the output voltage of a filter with frequency-response function $H(f)$ is

$$|s_0(t)| = \left|\int_{-\infty}^{\infty} S(f)H(f)\exp(j2\pi ft)\,df\right| \tag{5.8}$$

where $S(f)$ is the Fourier transform of the input (received) signal. The mean output noise power is

$$N = \frac{N_0}{2}\int_{-\infty}^{\infty} |H(f)|^2\,df \tag{5.9}$$

where N_0 is the input noise power per unit bandwidth. The factor 1/2 appears before the integral because the limits extend from $-\infty$ to $+\infty$, but N_0 is defined as the noise power per unit bandwidth *only* over positive values of f.

Substituting Eqs. (5.8) and (5.9) into (5.7), and letting t_m denote the time t at which the output $|s_0(t)|^2$ is a maximum, the ratio R_f becomes

$$R_f = \frac{\left|\int_{-\infty}^{\infty} S(f)H(f)\exp(j2\pi ft_m)\,df\right|^2}{\frac{N_0}{2}\int_{-\infty}^{\infty}|H(f)|^2\,df} \tag{5.10}$$

Schwartz's inequality states that if P and Q are two complex functions, then

$$\int P^*P\,dx \int Q^*Q\,dx \geq \left|\int P^*Q\,dx\right|^2 \tag{5.11}$$

The equality sign applies when $P = kQ$, where k is a constant. Letting

$$P^* = S(f)\exp(j2\pi ft_m) \quad \text{and} \quad Q = H(f)$$

and recalling that $\int P^*P\,dx = \int |P|^2\,dx$, application of the Schwartz inequality to the numerator of Eq. (5.10) gives

$$R_f \leq \frac{\int_{-\infty}^{\infty}|H(f)|^2\,df \int_{-\infty}^{\infty}|S(f)|^2\,df}{\frac{N_0}{2}\int_{-\infty}^{\infty}|H(f)|^2\,df} = \frac{\int_{-\infty}^{\infty}|S(f)|^2\,df}{\frac{N_0}{2}} \tag{5.12}$$

Parseval's theorem, which relates the energy in the frequency domain and the energy in the time domain, states that

$$\int_{-\infty}^{\infty}|S(f)|^2\,df = \int_{-\infty}^{\infty}|s(t)|^2\,dt = \text{signal energy} = E \tag{5.13}$$

Therefore,

$$R_f \leq \frac{2E}{N_0} \qquad [5.14]$$

which states that the output peak-signal-to-mean-noise ratio from a matched filter depends only on the total energy of the received signal and the noise power per unit bandwidth. It does not depend explicitly on the shape of the signal, its duration, or bandwidth; hence, these characteristics of a signal can be used to achieve radar capabilities other than signal detectability.

The frequency response function which maximizes the peak-signal-to-mean-noise ratio R_f is obtained by noting that the equality sign in Eq. (5.11) applies when $P = kQ$, or

$$H(f) = G_a S^*(f) \exp(-j2\pi f t_m) \qquad [5.15]$$

where the constant k has been set equal to $1/G_a$.

The matched filter has the interesting property that no matter what the shape, time duration, or bandwidth of the input-signal waveform, the maximum ratio of the output peak-signal-power-to-mean-noise power is simply twice the energy E contained in the received signal divided by the noise power per unit bandwidth N_0. The noise power per hertz of bandwidth is equal to kT_0F_n, where k in this case is the Boltzmann constant, T_0 is the standard temperature (290 K), and F_n is the receiver noise figure.

The concept of the matched filter assumes that the input signal is of the same form $s(t)$ as the transmitted signal (except for a difference in amplitude). This requires that the shape of the transmitted signal not change on reflection by the target or by propagation through the atmosphere. It also requires that the radial dimension of the target be small compared to the range resolution of the radar.

Output Signal from the Matched Filter From linear filter theory the output $y_0(t)$ of a filter is the convolution of the input $y_{in}(t) = s(t) + n(t)$ and the filter's impulse response $h(t)$, where $s(t)$ is the input signal and $n(t)$ is the input noise. It may be written as

$$y_0(t) = \int_{-\infty}^{\infty} y_{in}(\lambda) h(t - \lambda)\, d\lambda \qquad [5.16]$$

It was found previously that the impulse response of a matched filter is $h(t) = s(-t)$. (Here, for convenience, $G_a = 1$ and $t_m = 0$.) Then $h(t - \lambda) = s(-t + \lambda)$ and Eq. (5.16) becomes

$$y_0(t) = \int_{-\infty}^{\infty} y_{in}(\lambda) s(\lambda - t)\, d\lambda \qquad [5.17]$$

It is seen that the output of a matched filter as given by Eq. (5.17) is the cross correlation between the received signal $y_{in}(t)$ and the signal $s(t)$ that was transmitted, since the cross-correlation function between two signals $y_1(t)$ and $y_2(t)$ is defined as

$$\Phi(t) = \int_{-\infty}^{\infty} y_1(\lambda) y_2(\lambda - t)\, d\lambda \qquad [5.18]$$

When the signal-to-noise ratio is large, $y_{in}(t) \approx s(t)$, and the output signal from the matched filter is approximated by the autocorrelation function of the transmitted signal $s(t)$.

Figure 5.2 illustrates, in a highly simplified manner, the nature of the matched filter for a perfectly rectangular pulse of sinewave when the signal-to-noise ratio is large. In Fig. 5.2 (a) is the input signal; (b) is the frequency response function of the matched filter; (c) is the output of the matched filter (in the IF); and (d) is the envelope of the output of the matched filter, which is what appears in the video portion of the receiver.

Correlation Receiver Since the output of the matched filter is the cross-correlation function of the received signal and the transmitted signal, it is possible to implement the matched filter as a correlation process based on Eq. (5.17). In a correlation receiver the input signal $y_{in}(t)$ is multiplied by a delayed replica of the transmitted signal $s(t - T_R)$, where T_R is an estimate of the time delay of the target echo signal. The product is passed through a low-pass filter to perform the integration. If the output of the integrator (filter) exceeds a predetermined threshold at a time T_R, a target is said to be at a range $R = cT_R/2$, where c is the velocity of propagation. The cross-correlation receiver tests for the presence of a target at only a single time delay T_R. Targets at other time delays, or ranges, are found by either varying the value of T_R on successive transmissions or employing multiple channels and simultaneously performing the correlation process at all possible values of T_R. The need to search through all possible values of T_R can seriously complicate the correlation receiver.

Since the cross-correlation receiver and the matched-filter receiver are equivalent mathematically, the choice as to which to use in a particular radar application is determined by which is more practical to implement. The matched-filter receiver has almost always been preferred over the correlation receiver.

Approximation to the Matched Filter for a Rectangular-like Pulse The early radar pioneers in the 1930s were not aware of the concept of the matched filter; yet they learned from experience how to maximize the output signal-to-noise ratio for the simple pulse waveforms that were used at that time. They found that if the receiver passband was too wide compared with the spectral bandwidth of the radar signal, extra noise was introduced (since noise power is proportional to bandwidth); and the signal-to-noise ratio was reduced. On the other hand, if the receiver bandwidth was too narrow, the noise was

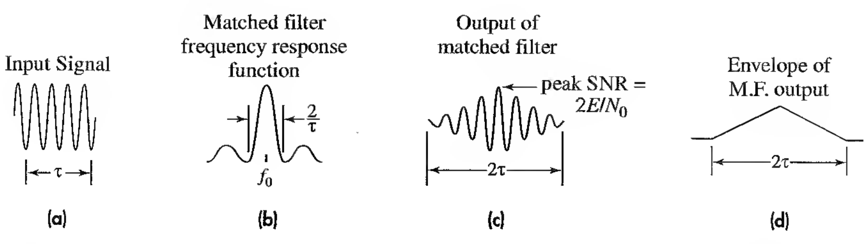


Figure 5.2 (a) Sketch of a perfectly rectangular pulse of sinewave of width τ and frequency f_0; (b) frequency-response function of the matched filter, where $H(f) = S^*(f) = S(f)$; (c) output of the matched filter; (d) envelope of matched-filter output.

reduced but so was the signal energy. Consequently, too narrow a bandwidth relative to the signal spectral width reduced the signal-to-noise ratio, and too wide a bandwidth also reduced the signal-to-noise ratio. Thus there was an optimum value of bandwidth relative to signal spectral width that maximized the signal-to-noise ratio. With rectangular-like pulses and conventional filter design, experience showed that the maximum signal-to-noise ratio occurred when the receiver bandwidth B was approximately equal to the reciprocal of the pulse width τ, or when $B\tau \approx 1$.

In practice the matched filter cannot be perfectly implemented. There will usually be some loss in signal-to-noise ratio compared to that of a theoretically perfect matched filter. The measure of efficiency is taken as the peak-signal-to-mean-noise ratio from the nonmatched filter divided by the peak-signal-to-noise ratio ($2E/N_0$) obtained from a matched filter. Table 5.1 lists values of $B\tau$ that maximize the signal-to-noise ratio (SNR) for various combinations of hypothetical filters and pulse shapes.[4,5] Note that the rectangular pulse assumed in Table 5.1 is not a realistic waveform since it has zero rise time, which implies infinite bandwidth. Radar pulses are bandwidth limited, and the rise time is approximately $1/B$. Also, several of the filters in Table 5.1 are not likely to be used in practice. Nevertheless, Table 5.1 is offered as an example of the performance of nonmatched filters. The usual "rule of thumb" when no other information is available, is to assume that a practical approximation to a matched filter has $B\tau \approx 1$ and a loss in SNR of about 0.5 dB.

Matched Filter for Nonwhite Noise In the derivation of the matched-filter characteristic [Eq. (5.15)], it was assumed that the spectrum of the input noise accompanying the signal was white; that is, it was independent of frequency. When this assumption does not

Table 5.1 Efficiency of nonmatched filters compared with matched filters

Input signal	Filter	Optimum $B\tau$	Loss in SNR, dB
Rectangular pulse	Third-order Bessel filter	0.78	0.47
Rectangular pulse	Quadruply tuned (Butterworth)	1.06	0.48
Rectangular pulse	Double tuned (Butterworth)	0.81	0.46
Rectangular pulse	5 cascaded single-tuned stages	0.67	0.51
Rectangular pulse	2 cascaded single-tuned stages	0.61	0.56
Rectangular pulse	Single tuned	0.40	0.88
Rectangular pulse	Rectangular	1.37	0.85
Rectangular pulse	Gaussian	0.74	0.51
Gaussian pulse	Rectangular	0.74	0.51
Gaussian pulse	Gaussian	0.44	0 (matched)

hold and the noise is represented by a nonwhite power spectrum $[N_i(f)]^2$, the frequency-response function that maximizes the peak-signal-to-mean-noise power has been shown to be[6,7]

$$H(f) = \frac{G_a S^*(f) \exp(-j2\pi f t_m)}{[N_i(f)]^2} \qquad [5.19]$$

This is the frequency response function of the *nonwhite-noise matched (NWN) filter.* When the noise is white, $[N_i(f)]^2$ = constant, and Eq. (5.19) reduces to that of Eq. (5.15) derived assuming white noise.

Equation (5.19) for nonwhite noise can be rewritten as

$$H(f) = \frac{1}{N_i(f)} \times G_a \left(\frac{S(f)}{N_i(f)}\right)^* \exp(-2\pi f t_m) \qquad [5.20]$$

From this the nonwhite-noise matched filter can be interpreted as the cascade of two filters. The first filter, with frequency-response function $1/N_i(f)$, makes the noise spectrum uniform, or white. It is sometimes called the *whitening filter.* The second is the matched filter given by Eq. (5.15) when the input noise is white and the signal spectrum is $S(f)/N_i(f)$.

It is seldom that noise is nonuniform over the bandwidth of the radar receiver. The nonwhite-noise matched filter is interesting, but it has had almost no application in radar.

Summary of the Matched Filter The characteristics of the matched filter for an input signal $s(t)$ are summarized below in short notation, omitting realizability factors and constants. The symbols have been defined previously in this section.

1. Frequency response function: $S^*(f)$
2. Maximum output signal-to-noise ratio: $2E/N_0$
3. Magnitude of the frequency response: $|H(f)| = |S(f)|$
4. Phase of the frequency response: $\phi_m(f) = -\phi_s(f)$
5. Impulse response: $s(-t)$
6. Output signal waveform for large signal-to-noise ratio: autocorrelation function of $s(t)$
7. Relation between bandwidth and pulse width for a rectangular-like pulse and conventional filter: $B\tau \approx 1$
8. Frequency response function for nonwhite noise: $S^*(f)/[N_i(f)]^2$

The matched filter makes radar-signal detection quite different from detection in conventional communication systems. The detectability of signals with a matched-filter receiver is a function only of the received signal energy E and the input noise spectral density N_0. Detection capability and the range of a radar do not depend on the shape of the signal or the receiver bandwidth. The shape of the transmitted signal and its bandwidth therefore can be selected to optimize the extraction of information without, in theory, affecting detection. Also different from communications is that the signal out of the matched filter is not the same shape as the input signal. It should be no surprise that the output

signal's shape is different from the input since the criterion for the matched filter states only that detectability is to be maximized, not that the shape of the signal is to be preserved.

5.3 DETECTION CRITERIA

Detection of signals is equivalent to deciding whether the receiver output is due to noise alone or to signal plus noise. This is the type of decision made (probably subconsciously) by a human operator from the information presented on a radar display. When the detection process is carried out automatically by electronic means without the aid of an operator, the detection criterion must be carefully specified and built into the decision-making device.

In Chap. 2, the radar detection process was described in terms of threshold detection. If the envelope of the receiver output exceeds a pre-established threshold, a signal is said to be present. The threshold level divides the output into a region of no detection and a region of detection. The radar engineer selects the threshold that divides these two regions so as to achieve a specified probability of false alarm, which in turn is related to the average time between false alarms. The engineer then determines the other parameters of the radar needed to obtain the signal-to-noise ratio for the desired probability of detection.

Neyman-Pearson Observer The usual procedure for establishing the decision threshold at the output of the radar receiver is based on the classical statistical theory of the *Neyman-Pearson observer.* This is described in terms of the two types of errors that might be made in the detection decision process.

One type of error is to mistake noise for signal when only noise is present. It occurs whenever the noise out of the receiver is large enough to exceed the decision-threshold level. In statistics this is called a Type I error. In radar it is a *false alarm.* A Type II error occurs when a signal is present, but is erroneously considered to be noise. The radar engineer would call such an error a *missed detection.* It might be desired to minimize both errors, but they both cannot be minimized independently. In the Neyman-Pearson observer, the probability of a Type I error is fixed, and the probability of a Type II error is minimized.

As discussed in Sec. 2.5, the threshold level is set by the radar engineer so that a specified false-alarm probability is not exceeded. This is equivalent to fixing the probability of a Type I error and minimizing the Type II error (or maximizing the probability of detection), which is the Neyman-Pearson test used in statistics for determining the validity of a specified statistical hypothesis.[8] In statistical terms it is claimed to be a uniformly most powerful test and an optimum one, no matter what the a priori probabilities of signal and noise. The Neyman-Pearson criterion is employed in most radars for making the detection decision, whether knowingly or not.

Likelihood-Ratio Receiver The *likelihood ratio* is a statistical concept that has been used in radar detection theory and information extraction theory to model optimum decision

procedures. It is defined as the ratio of two probability density functions, with and without signal present, or

$$L_r(v) = \frac{p_{sn}}{p_n} \tag{5.21}$$

where p_{sn} is the probability-density function for signal plus noise and p_n is the probability-density function for noise alone. In Chap. 2 these two probability-density functions were given by Eqs. (2.27) and (2.21), respectively. The likelihood ratio is a measure of how likely it is that the envelope v of the receiver output is due to signal plus noise as compared with noise alone. If the likelihood ratio is sufficiently large, it would be reasonable to conclude that a signal is present.

The Neyman-Pearson observer is equivalent to examining the likelihood ratio and determining if $L_r(v) \geq K$, where K is a real, nonnegative number that depends on the probability of false alarm selected.

One does not find likelihood-ratio receivers in equipment catalogs. It is a statistical concept that models the basic nature of a receiver for maximizing the detectability of radar signals or a receiver that provides the most accurate measurement of radar parameter (such as range). The likelihood ratio is an analytical tool used to indicate optimum receiver and detector design. In most cases of practical interest a radar that employs a matched filter is equivalent to a likelihood-ratio receiver.[8]

Inverse Probability Receiver This is another statistical concept and is based on the relationship known as Bayes' rule for the probability of causes.[9,10] As with the likelihood ratio, inverse probability has been used as an analytical basis to model "optimum" receivers for detection and information extraction. *Inverse probability* is different from the more familiar *direct probability* that describes the chance of an event happening on a given hypothesis. If an event actually happens (such as a voltage appearing at the output of the radar receiver), the problem of forming the best estimate of its cause is a problem in inverse probability.

The operation of the inverse probability receiver will not be described here. (More detail can be found in earlier editions of this text and in the references thereto.) The inverse probability receiver, likelihood receiver, and matched-filter receiver, however, are all related to one another under certain conditions which are often met in most radar applications. The design information obtained from one can usually be obtained from the others as well. The inverse probability receiver differs from the likelihood-ratio receiver (and the matched filter) in that it requires knowledge of a priori probabilities. (An a priori probability is one that is known before the event occurs; e. g., the a priori probability that a flip of a coin results in a heads is 0.5.) It is not usually possible in radar to define quantitatively the a priori probability (for example, the probability of observing an aircraft echo signal at the output of the radar at a range of 110 nmi, azimuth of 75°, at 0630 tomorrow morning). Therefore, the inverse probability receiver has only been of academic interest. Sometimes it has been suggested that the problem of selecting the a priori probability can be satisfied by assuming it to be constant. If the a priori probability is constant, the inverse probability receiver reduces to the likelihood-ratio receiver. Thus one might as well start with the likelihood-ratio receiver in the first place.

The inverse probability receiver and the likelihood-ratio receiver are statistical models that have been used in the past to derive important relations in the theory of signal detection and information extraction. Although one does not build either type of receiver, they both have been useful since theoretical results derived from them indicate the best that can be achieved under the given assumptions.

Sequential Observer, Sequential Detection In a conventional radar based on the Neyman-Pearson observer, a fixed number of pulses, n, are obtained before a detection decision is made. When the signal-to-noise ratio is large, it might not be necessary to collect all n pulses before being able to make the decision that a target echo signal is present. Also, it might be possible to determine after only a few pulses that the receiver output is so low it is unlikely that, even with the remaining pulses, the integrated receiver output would cross the threshold. It should be possible, by taking advantage of the possibility of a quick decision, to make a detection decision with fewer pulses, on average, than would be needed for the Neyman-Pearson observer. This procedure is called the *sequential observer.*[11,12] It is an interesting detection method that, in some cases, can result in almost an order of magnitude decrease in power or revisit time when it can be applied. Unfortunately, its application in radar is limited.

After a single sample of the receiver output, the sequential observer makes one of three choices: (1) the sample is due to the presence of signal and noise; (2) it is due to noise alone; or (3) it cannot be determined whether it is due to noise alone or signal-plus-noise. If it can be decided that either No. 1 or No. 2 applies, the test is completed and the radar moves to the next resolution cell to repeat the operation. If the choice is No. 3, a decision cannot be made, and another observation is obtained and the choices examined again on the basis of the two observations. This procedure is repeated until a decision can be made as to whether noise alone or signal-plus-noise is present.

The sequential observer fixes the probability of errors beforehand and allows the number of observations (integration time) to vary. This procedure theoretically allows a significant reduction in the average number of pulses (samples) needed for making a decision. The sequential observer makes a relatively prompt decision when only noise is present. In one reported example,[13] the sequential observer can, on average, come to a decision with less than one-tenth the number of observations required for the Neyman-Pearson observer when only noise is present. When a threshold signal is present, the sequential observer requires, on average, about one-half the number of observations of the equivalent fixed-sample Neyman-Pearson observer.

A flexible phased array radar, or equivalent agile antenna, is required for the sequential observer in order to take advantage of the variable number of pulses to be integrated. Unfortunately, there is a severe limitation to its use. If there is only one range cell in each angular resolution cell, such as a "guard band," the sequential observer works as indicated above. At each angular position of a surveillance radar antenna, however, there can be a large number of range cells. The sequential observer must come to a decision in every one of these cells before moving on to the next angular position. Any savings in time to make a decision is lost when the number of cells is large, since the observation time at any angular position is determined by the time it takes for the slowest cell to make a decision.[12]

Although the sequential observer can, in principle, result in a saving in transmitter power or in revisit time, it is limited to applications such as a guard ring, the detection of border penetration,[14] or a radar with an omnidirectional transmit antenna and many contiguous fixed narrow-beam receiving antennas that look everywhere all the time.

The term *sequential detection* is sometimes used synonymously with the term sequential observer; but it is also used to describe a two-stage detection process that can be employed with a phased-array radar.[15,16] The radar transmits a pulse or a series of pulses in a particular direction, but with a lower threshold (and higher false-alarm probability) than normal. If no threshold crossings are obtained, the antenna beam moves to the next position. If a threshold crossing occurs, a second pulse or series of pulses is transmitted with higher energy, and with a higher threshold. A detection is declared if the threshold is crossed on both transmissions. It has been claimed that a second threshold is employed in about 4 percent of the beam positions and that there is a power saving of from 3 to 4 dB as compared with uniform scanning.

5.4 DETECTORS

The detector is that portion of the radar receiver that extracts the modulation from the carrier in order to decide whether or not a signal is present. It extends from the IF amplifier to the output of the video amplifier; thus, it is much more than a rectifying element. The conventional pulse radar as described in Chaps. 1 and 2 employs an *envelope detector* which extracts the amplitude modulation and rejects the carrier. By eliminating the carrier and passing only the envelope, the envelope detector destroys the phase information. There are other "detectors" in radar that are different from the above description. The MTI radar uses a *phase detector* to extract the phase of the radar echo relative to the phase of a coherent reference, as described in Chap. 3. In Chap. 4, the *phase-sensitive detector* employed in tracking radars for extracting angle information was mentioned.

Optimum Envelope Detector Law The envelope detector consists of the IF amplifier with bandpass filter characteristic, a rectifying element (such as a diode), and a video amplifier with a low-pass filter characteristic. The detector is called a *linear detector* if the relation between the input and output signal is linear for positive voltage signals, and zero for negative voltage. (The detector, of course, is a nonlinear device even though it bears the name *linear.*) When the output is the square of the input for positive voltage, the detector is called *square law.* The detector law is usually considered the combined law of the rectifying element and the video integrator that follows it, if an integrator is used. For example, if the rectifying element has a linear characteristic and the video integrator has a square-law characteristic, the combination would be considered a square-law detector. There can be, of course, many other detector laws beside the linear and the square law.

The *optimum detector* law can be found based on the use of the likelihood-ratio receiver. It can be expressed as[17–19]

$$y = ln\ I_0(av) \tag{5.22}$$

where y = output voltage of the detector

a = amplitude of the sinewave signal divided by the rms noise voltage

v = amplitude of the IF voltage envelope divided by the rms noise voltage

$I_0(x)$ = modified Bessel function of zero order

This equation specifies the form of the detector law that maximizes the likelihood ratio for a fixed probability of false alarm. A suitable approximation is[20]

$$y = ln\ I_0(av) \approx \sqrt{(av)^2 + 4} - 2 \qquad [5.23]$$

For large signal-to-noise ratios (a $\gg$ 1), this is approximately

$$y \approx av$$

which is a linear law. For small signal-to-noise ratios, the approximation of Eq. (5.23) becomes

$$y \approx (av)^2/4$$

which has the characteristic of a square-law detector. Hence, for large signal-to-noise ratio, the optimum $ln\ I_0$ detector may be approximated by a linear detector, and for small signal-to-noise ratios it is approximated by a square-law detector.

The linear detector usually is preferred in practice since it results in a higher dynamic range than the square law and is less likely to introduce distortion. On the other hand, the square-law detector usually is easier to analyze than the linear, so many analyses assume a square-law characteristic. Fortunately, the theoretical difference in detection performance between the square-law and linear detectors when performing noncoherent integration often is relatively insignificant.[21,22] Marcum[23] also showed that for a single pulse (no integration) the probability of detecting a given signal is independent of the detector law.

Logarithmic Detector If the output of the receiver is proportional to the logarithm of the input envelope, it is called a *logarithmic detector,* or *logarithmic receiver.* It finds application where large variations of input signals are expected. Its purpose is to prevent receiver saturation and/or to reduce the effects of unwanted clutter echoes in certain types of non-MTI receivers (as in the discussion of the log-FTC receiver in Sec. 7.8). A logarithmic characteristic is not used with MTI receivers since a nonlinear characteristic can limit the MTI improvement factor that can be achieved.

There is a loss in detectability with a logarithmic receiver. For 10 pulses integrated the loss in signal-to-noise ratio is about 0.5 dB, and for 100 pulses integrated, the loss is about 1.0 dB.[24] As the number of pulses increase, the loss approaches a maximum value of 1.1 dB.[25]

***I,Q* Detector** The *I* and *Q,* or *in-phase* and *quadrature,* channels were mentioned in Sec. 3.5 in the discussion of the MTI radar. There it was noted that a single phase-detector fed by a coherent reference could produce a significant loss in signal depending on the relative timing (or "phase") of the pulse train and the doppler-shifted echo signal. In an MTI radar, the term *blind phase* (not a truly descriptive term) was used to describe this loss.

The loss due to blind phases was avoided if a second parallel detector channel, called the *quadrature,* or Q channel, were used with a reference signal 90° out of phase from the reference signal of the first channel, called the *in-phase,* or I channel. Most signal processing analyses now use I and Q channels as the receiver model especially when the doppler frequency is extracted.

The I,Q detector is more general than just for avoiding loss due to blind phases in an MTI radar. Figure 5.3 illustrates the I,Q detector. It is sometimes called a *synchronous detector.*[26] If the input is a narrowband signal having a carrier frequency f_0 (which could be the IF frequency) with a time-varying amplitude $a(t)$ and time-varying phase $\phi(t)$, then

$$\text{input signal: } s(t) = a(t) \sin [2\pi f_0 t + \phi(t)]$$

The output of the in-phase channel is $I(t) = a(t) \cos [\phi(t)]$ and the output of the quadrature channel is $Q(t) = a(t)\sin [\phi(t)]$. The input signal then can be represented as $s(t) = I(t) \sin 2\pi f_0 t + Q(t) \cos 2\pi f_0 t$. Thus the I and Q channels together provide the amplitude and phase modulations of the input signal.

If the outputs of the I and Q channels of Fig. 5.3 are squared and combined (summed), then the square root of the sum of the squares is the envelope $a(t)$ of the input signal. This describes an envelope detector. The phase $\phi(t)$ of the input signal is arctan (Q/I).

The I,Q representation is commonly used in digital signal processing.[27] The digitized signals are represented by complex numbers derived from the I and Q components. In each channel, the signal is digitized by an analog-to-digital (A/D) converter to produce a series of complex digital samples from the signal $I + jQ$. According to the sampling theorem, if the input signal has a bandwidth B there must be at least $2B$ samples per second (the Nyquist rate) to faithfully reproduce the signal. Because there are two channels in the I,Q detector, the A/D converter in each of the I and Q channels needs only to sample at the rate of B samples per second, thus reducing the complexity required of the A/D converters.

With a rate of B samples per second, there is a loss of about 0.6 dB compared to continuous sampling, since the sampling is not guaranteed to occur at the peak of the

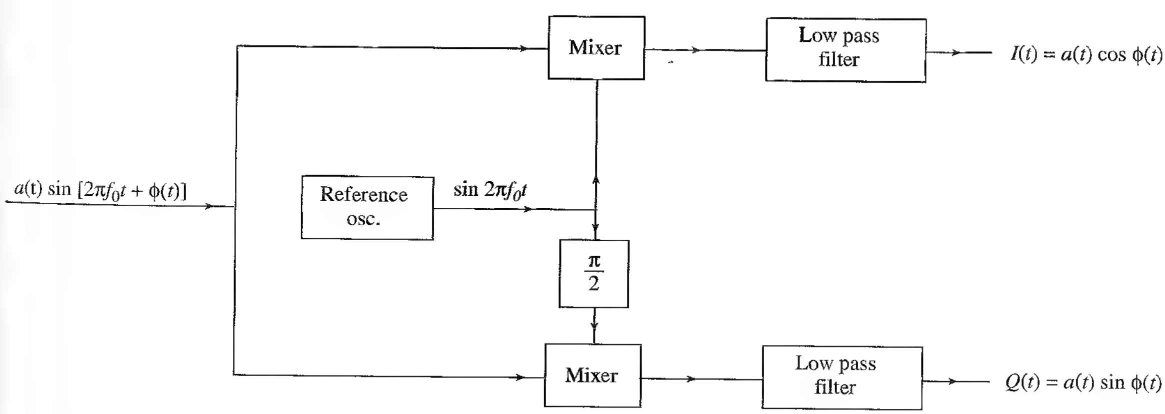


Figure 5.3 *I,Q* detector

output.[27] Much of this loss can be recovered by sampling at a rate of $2B$ samples per second. In some applications, further loss might occur due to the two channels not being precisely 90° out of phase, not being of equal gain, or if they are not perfectly linear.[28]

When I,Q channels are used for MTI processing, a doppler filter such as a delay-line canceler is included in each channel to separate moving targets from stationary clutter, as was discussed in Sec. 3.5.

Coherent Detector The so-called "coherent detector" sometimes has been described in the past literature as a single-channel detector similar to the in-phase channel of the I,Q detector, but with the reference signal at the same exact frequency and same exact phase as that of the input signal. Compared to the normal envelope detector of Chap. 2, the signal-to-noise ratio from a coherent detector might be from 1 to 3 dB greater. Unfortunately, the phase of the received radar signal is seldom known, so the single-channel coherent detector as described generally is not applicable to radar. The I,Q detector of Fig. 5.3 can also be considered as a coherent detector, but without the limitation of the coherent detector described above.

5.5 AUTOMATIC DETECTION

An operator viewing a PPI display or an A-scope "integrates" in his/her eye-brain combination the echo pulses available from the target. Although an operator in many cases can be as effective as an automatic integrator, performance is limited by operator fatigue, boredom, overload, and the integrating characteristics of the phosphor of the CRT display. With automatic detection by electronic means, the operator is not depended on to make the detection decision. *Automatic detection* is the name applied to the part of the radar that performs the operations required for the detection decision without operator intervention. The detection decision made by an automatic detector might be presented to an operator for action or to a computer for further processing.

In many respects, automatic detection requires much better receiver design than when an operator makes the detection decision. Operators can recognize and ignore clutter echoes and interference that would limit the recognition abilities of some automatic devices. An operator might have better discrimination capabilities than automatic methods for sorting clutter and interference; but the automatic, computer-based decision devices can operate with far greater number of targets than an operator can handle.

Automatic detection of radar signals involves the following:

- *Quantization* of the radar coverage into range, and maybe angle, resolution cells.
- *Sampling* of the output of the range-resolution cells with at least one sample per cell, more than one sample when practical.
- *Analog-to-digital conversion* of the analog samples.
- *Signal processing* in the receiver to remove as much noise, clutter echoes, and interference as practicable before the detection decision is attempted.
- *Integration* of the available samples at each resolution cell.

- *Constant false-alarm rate* (CFAR) circuitry to maintain the false-alarm rate when the receiver cannot remove all the clutter and interference.
- *Clutter map* to provide the location of clutter so as to ignore known clutter echoes.
- *Threshold detection* to select target echoes for further processing by an automatic tracker or other data processor.
- *Measurement of range and angle* after the detection decision is made.

The automatic detection and tracking (ADT) system, which includes the above, was discussed in Sec. 4.9. We next consider the automatic integration of signals and the application of CFAR in the automatic detection process.

5.6 INTEGRATORS

A major part of an automatic detector that operates with more than one pulse is the *integrator* which integrates, or adds, the energy from the received pulses available from a target. The subject of predetection and postdetection integration was introduced in Sec. 2.6. In this section, several integration methods will be briefly reviewed. Integration of pulses in early radars often was performed by an operator viewing a cathode-ray tube display since automatic integration of pulses was seldom practical with the then existing analog technology. Modern radars almost always implement the integration of signals digitally. (Note that in the technical literature, some integration devices are called *detectors* even though they perform *integration.*)

Moving-Window Integrator[29] The straightforward method for integrating the n pulses available from a target is to simply add them. It was not until advances in digital processing technology became available, however, that it became practical to do so. Continuous integration of the last n pulses at the output from a receiver from each range-resolution cell can be accomplished with a *moving-window integrator,* also called *moving-window detector.* The new output from the receiver is added to the previous sum, and the output received n pulses earlier is subtracted to achieve a running sum of n pulses. In a digital processor it is possible to apply weights to the outputs, based on the two-way gain of the antenna pattern, so as to provide increased signal-to-noise ratio. If uniform weighting is used instead (since it is easier to do), there is a loss in signal-to-noise ratio of about 0.5 dB compared to optimum weighting.[30]

The angular location of the target may be estimated by taking the midpoint between the first and last crossings of the detection threshold or by noting the location of the maximum value of the running sum. After correcting for the bias, the accuracy of the angular location measurement is only about 20 percent worse than theoretical.[30]

According to Trunk,[31] a disadvantage of the moving-window detector is that it is susceptible to large interference signals, a problem that can be minimized by using limiting. It also requires large storage since the last n pulses from each range cell must be put in memory. With increasing improvements in digital technology, this limitation has become less of a concern.

Binary Integration This was the first automatic method developed to integrate pulses and make the detection decision without the aid of an operator.[32] It is still an important technique. Its chief advantage is that it can be implemented without the complexity of the moving-window integrator. It is, however, less efficient than ideal postdetection integration.

As a radar antenna scans by a target it will receive n echo pulses. If m of these expected n pulses exceed a predetermined value (threshold), a target is declared to be present. The use of a detection criterion that requires m out of n echo pulses to be present is a form of integration. It is called the *binary integrator,* but it is also well-known as the *binary detector, double-threshold detector,* and *m-out-of-n detector.*

A block diagram of the binary integrator is shown in Fig. 5.4. The radar video is passed through a threshold detector, whose level is lower than the normal threshold discussed previously in Chap. 2. It is the first of two thresholds in this system, hence the name *double-threshold detector.* The output of the first threshold is sampled by the quantizer at least once per range-resolution cell. A pulse with a standard amplitude is generated if the video waveform exceeds the first threshold, and nothing is generated if it does not exceed the threshold. These outputs are designated 1 and 0, respectively. Thus the output of the quantizer is a series of 1s and 0s The 1s and 0s from the last *n pulses* at each range cell are stored and counted in the binary counter. If there are at least m 1s within the last n sweeps, a target is said to be detected at that range. The number m is the second threshold to be passed in the double-threshold detector. The two thresholds must be selected jointly for best performance.

The optimum value of m/n for a nonfluctuating echo signal is shown in Fig. 5.5.[33] This curve is only approximate since there is a slight dependence on the false alarm probability, but it is said to be independent of the signal-to-noise ratio. A fluctuating Swerling Case 1 target has the same optimum value of m/n as a nonfluctuating target, but a fluctuating Swerling Case 2 has different optimum values.[34–36] The optimum value of m is not a sensitive selection. It can be quite different from the optimum without significant penalty.

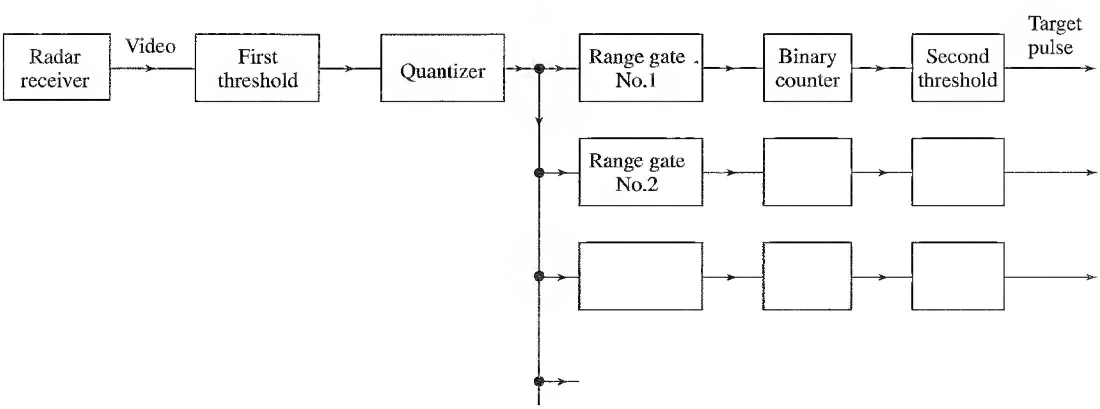


Figure 5.4 Block diagram of a binary integrator

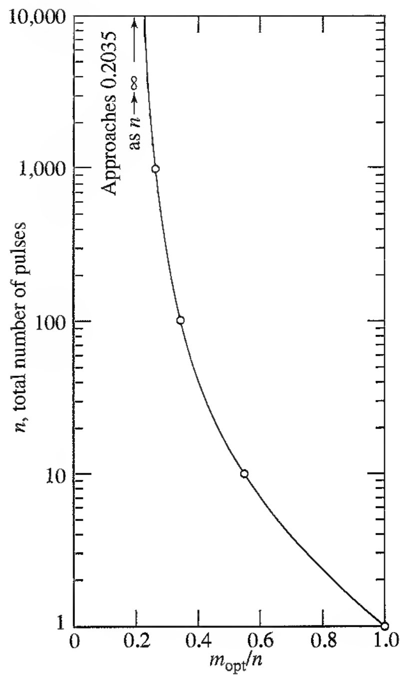


Figure 5.5 Optimum number of pulses m_{opt} (out of a maximum of n) for a binary moving window detector, assuming a constant (nonfluctuating) target echo. (After Swerling,[33] courtesy Rand Corp.)

The loss in signal-to-noise ratio due to quantizing signals into two levels (1 or 0) in the binary integrator can vary from just under 1 dB to 2.5 dB.[37] For the particular case given in ref. 37 where the probability of detection is 0.9 and the probability of false alarm is 10^{-6}, the loss can reach 1.4 dB for a nonfluctuating target and 2.2 dB for a Swerling Case 1 fluctuating target, when n is about 8 hits per target. For larger values of n (n greater than about 100) the binary integrator asymptotically approaches a loss of 0.94 dB in all cases, as compared with optimum noncoherent integration. When the amplitude is quantized into more than two levels, the loss is less. V. Gregers-Hansen[37] states that quantization into four levels (two bits) reduces the loss to about one-third that of the two-level quantization.

The binary integrator is relatively simple as an automatic detector and is less sensitive to the effects of a single large interference pulse that might exist along with the target echo pulses. In the conventional integrator, the full energy of the interference pulse is included in the sum. This could result in a false-target indication even though only one interference pulse were present. In a binary integrator, however, a large interference pulse contributes no more to the sum than would any other pulse that crosses the first threshold. No matter what the energy in the pulse, the output of the first threshold is a "1". The same advantage occurs when the background is not receiver noise, but is nongaussian (or non-Rayleigh) clutter as in high-resolution sea clutter and many forms of land clutter. If the clutter statistics have high tails (which means a higher probability of having large values than the gaussian probability density function), these high values can result in false-target reports when a detection is based on gaussian statistics. The binary integrator treats

these high values of clutter as any other first-threshold crossing, and it is not as likely to report a target when none is present as might a conventional detection criterion based on the total energy received in n pulses. The binary integrator is therefore *robust,* in that it can be used when the background noise or clutter is nongaussian.

An estimate of the target's angular position (beam splitting) also may be made by locating the center of the group of n pulses. For large n, the angular estimation error made with the binary integrator is about 25 percent greater than the theoretical lower bound.[31]

Batch Integrator[31] A *batch integrator* is used when there is a large number of pulses available. If there are kn pulses received from the target, k pulses are summed (batched) and compared to a threshold to make a binary decision (0 or 1) as to whether the threshold has been crossed. The process is repeated for each of the remaining $n - 1$ sets of k pulses. The n 0s and 1s are summed and compared to a second threshold. The batch integrator, just as the binary integrator, is simpler to implement, is less affected by interference spikes, and is robust to the noise or clutter statistics. It is said to require less storage, have better detection performance, and provide a more accurate angle estimation than the binary integrator.

Feedback Integrators[31] The advantage of the single delay-line feedback integrator is its simpler processing. As indicated in Fig. 5.6a, in this integrator the output of the delay line is recirculated so that the signals from each new sweep are added to the sum of all the previous sweeps. To prevent unwanted oscillations ("ringing") due to the positive feedback, the sum must be attenuated by an amount $k < 1$ after each pass through the delay line. The factor k is the gain of the loop formed by the delay line and the feedback path. It imparts an exponential weighting to the received pulses. The effective number of pulses integrated is equal to $(1 - k)^{-1}$.

The single delay-line feedback integrator has a loss of about 1.0 dB in signal-to-noise ratio compared to the ideal postdetection integrator that weights the received pulses in

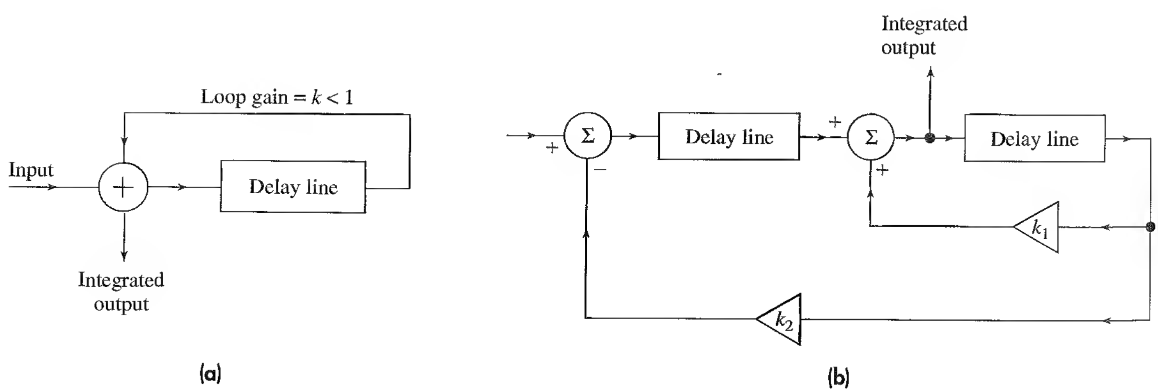


Figure 5.6 Recirculating delay-line integrotor, or feedback integrator, k = loop gain < 1. (a) Single deloy loop; (b) two-pole filter.

proportion to the two-way antenna gain. Estimating the angular location of the target based on threshold crossings produces a 20 percent error compared to the optimum. There is a bias, however, that must be estimated, and it can be large. The single delay-line integrator of Fig. 5.6a might have the advantage of simplicity, but its problems cause it to have only limited utility.

The two-pole filter of Fig. 5.6b requires more storage than the single delay-line feedback integrator, but its detection performance is only 0.15 dB less than the optimum. Its angular measurement accuracy has a standard deviation 15 percent greater than optimum, and the estimator based on the maximum value has a constant bias.[38] According to Trunk[31] the problems with this integrator are that it has rather high detector sidelobes (15 to 20 dB) and it is extremely sensitive to interference.

Other Types of Integrators/Detectors Some forms of integrators are also called detectors because the detection decision uses the n pulses received from a target. The *mean detector,* for example, is one that sets a threshold based on the mean, or average, of its n received pulses. It is therefore equivalent to the conventional method of setting the threshold on the basis of the addition (integration) of n pulses. The *median detector* sets a threshold based on the median value of its n expected pulses. It is more robust than a mean detector in that it is not as adversely affected by a large interference pulse that might be included among the n pulses. Also, it is not as degraded as the mean detector when the clutter or noise background is described by nongaussian statistics. There are also *censored mean detectors* in which one or more of the largest amplitude pulses of the n pulses received are eliminated from the detection decision on the assumption that they are likely to be from interference rather than from a target. The adaptive detector, nonparametric detector, and distribution-free detector are usually considered as forms of CFAR, which is discussed in the next section. Most of these detectors have been more of academic interest than candidates for application in operational radar systems.

5.7 CONSTANT-FALSE-ALARM RATE (CFAR) RECEIVER

As said before in this text, a target is detected when the output of the radar receiver crosses a predetermined fixed threshold level set to achieve a specified probability of false alarm. When noise at the receiver is due to internally generated noise of fixed level described by a gaussian probability density function, the procedure for setting the threshold is well established (Sec. 2.8). In many situations, however, clutter echoes and/or hostile noise jamming can be much larger than receiver internal noise. When this happens, the receiver threshold can be exceeded and many false alarms can occur, which cause havoc with radar detection and tracking.

A well-trained and alert operator viewing a PPI or other radar display is seldom misled into mistaking clutter or jamming for real targets, but an operator can lose effectiveness when there are many target echoes to be processed. An automatic detection and tracking (ADT) system can handle many targets, and will attempt to determine if clutter or jamming signals that cross the receiver threshold form realistic tracks. Eventually, a false alarm will not form a realistic track and will be discarded by the tracking computer. An

automatic system, however, might be of limited capability and require too much time or computer capacity to recognize and discard false alarms. Although digital computers can have a high level of capability, the task of recognizing false echoes might cause them to be overloaded when there are a large number of real targets, a large number of clutter echoes, interference, and/or high external noise levels. Therefore, if ADT is to work properly, some method is necessary to keep clutter and external noise from reaching the automatic-tracking computer. One method has been CFAR, or *constant false alarm rate* receiver. CFAR automatically raises the threshold level to keep clutter echoes and external noise from overloading the automatic tracker with extraneous information. The need for CFAR was recognized when the early automatic detection and tracking systems were installed as add-ons to existing radars with no MTI or relatively poor MTI that did not have good clutter rejection. CFAR is achieved, of course, at the expense of a lower probability of detection of desired targets. In addition, CFAR can also produce false echoes when there is nonuniform clutter, suppress nearby targets, and degrade the range resolution.

Cell Averaging CFAR, or CA CFAR The major form of CFAR has been the cell-averaging CFAR, due to Finn and Johnson,[39] and its variants. It is illustrated in Fig. 5.7. It uses an adaptive threshold whose level is determined by the clutter and/or noise in the vicinity of the radar echo. Two tapped delay-lines sample echo signals from the environment in a number of *reference cells* located on both sides of the test cell (the range cell of interest). The spacing between reference cells is equal to the radar range resolution (usually the pulse width). The output of the test cell is the radar video output, which is compared to the adaptive threshold derived from the sum of the outputs of the tapped delay lines defining the reference cells. This sum, therefore, represents the radar environment to either side of the test cell. It changes as the radar environment changes and as the pulse travels out in time. When multiplied by a predetermined constant k, the sum provides an adaptive threshold to maintain a constant false-alarm rate. Thus the threshold can adapt to the environment as the pulse travels in time.

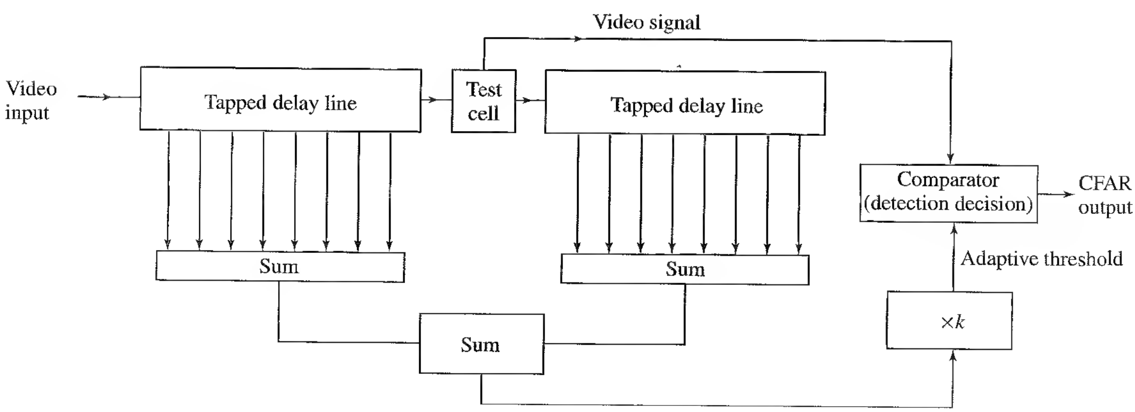


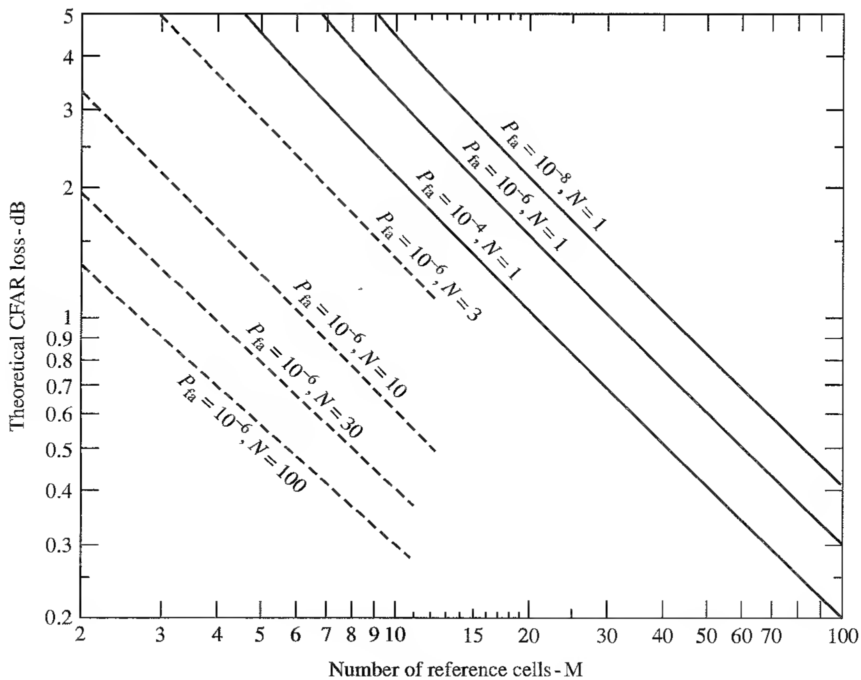
Figure 5.7 Cell averaging CFAR

If the radar output is noise or clutter described by the Rayleigh probability density function, the constant k which multiplies the sum of the tapped delay lines can be determined from classical detection theory, similar to that described in Chap. 2. When the statistics of the clutter are not known, which is often the case, the value of k can only be estimated or some form of nonparametric detector used.

CFAR Loss The greater the number of reference cells (delay-line taps) in the CA CFAR the better is the estimate of the background clutter or noise and the less will be the loss in detectability (signal-to-noise ratio). There is a limit, however, to the number of reference cells that can be used in practice since the clutter must be relatively homogeneous over the reference cells. A typical CFAR design for an aircraft-surveillance radar might have a total of 16 to 20 reference cells that sample the environment a half-mile to either side of the signal in the test cell. In a doppler processing radar, such as MTI or pulse doppler, reference cells can sometimes be taken from adjacent doppler filters as well as from adjacent range cells.

Since there are only a finite number of reference cells, the estimate of the noise or clutter is not precise and there will be a loss in detectability. Figure 5.8, derived from the publications of Mitchell and Walker[40] and from R. Nitzberg,[41] gives the theoretical CFAR loss as a function of the number of reference cells M, the probability of false alarm, and the number N of pulses integrated. (The CFAR loss is the signal-to-noise ratio required when CFAR is employed divided by the signal-to-noise ratio required for fixed-threshold

Figure 5.8 Theoretical CFAR loss. (After R. L. Mitchell and J. F. Walker[40] and R. Nitzberg.[41])



detection.) The solid curves apply for detection using only a single pulse. The dashed curves give the loss for a probability of false alarm of 10^{-6}, when the number of pulses N is greater than one. The curves of Fig. 5.8 apply to a nonfluctuating target as well as Swerling Case 1 and Case 2. When the number of reference cells is large, the CFAR loss is small. Nitzberg shows that the CFAR loss for single-pulse detection ($N = 1$) can be approximated by

$$\text{Loss (dB)} = -\frac{5}{M} \log P_{\text{fa}} \qquad [5.25]$$

where P_{fa} is the probability of false alarm.

Clutter Edges The CA CFAR of Fig. 5.7 assumes that the statistics of the clutter or noise at each reference cell are independent, identical, and the same as the statistics at the test cell. This is not the case at the edges of clutter. As the reference cells pass over the leading and trailing edges of a patch of clutter, not all the cells contain clutter; so the threshold will be lower than when all reference cells contain clutter. False alarms (threshold crossings), therefore, can result at clutter edges. Threshold crossings from the clutter edges can be reduced by summing the leading and the lagging references cells separately, and using the greater of the two to determine the threshold.[42] The CFAR that uses the *greater of* the two sets of reference cells is known as GO-CFAR. It introduces an additional CFAR loss of from 0.1 to 0.3 dB.[43]

Effect of Multiple Targets When there are one or more targets within the reference cells along with a primary target in the test cell, the threshold is raised (even in the absence of any clutter) and the detection of the primary target in the test cell of the CA CFAR might be suppressed. One method for reducing the effect of multiple targets is to remove (censor) the outputs of those reference cells that are much larger than the rest. A predetermined number J of reference cells (those with the largest outputs) are removed and the adaptive threshold determined by the outputs of the remaining $M - J$ cells.[44] This is known as a *censored mean-level detector* (CMLD). The number of censored cells should be equal to, or at least not smaller than, the number of interfering targets. The loss associated with the CLMD has been analyzed and has been said to be small,[45] but it can be 1 dB or greater.[46]

Another approach to handling multiple nearby targets is that of *ordered statistic,* or OS, CFAR, in which the CFAR threshold is determined from one single value selected from the so-called ordered statistic.[47,48] The outputs from M reference cells are put in order from smallest to largest, and the Kth ordered value when multiplied by a scalar is used to set the threshold. For example, if $M = 16$, K might be 10. In one particular analysis of the OS CFAR, the additional loss in signal-to-noise ratio is given as about 0.5 dB for one interfering target, and about 1 dB for two interfering targets.

The problem of dealing with the effect of additional targets within the reference cells has received much attention in the literature.[49–58] Still another method for dealing with loss in detectability when multiple targets are present within the reference cells is to employ log video in which a log detector is used ahead of a CA CFAR to suppress large echoes.[59]

Range Resolution In general, two equal-amplitude targets can be resolved if they are separated in range by about 0.8 pulse width. The usual CFAR, however, considerably degrades the range resolution so that two equal-amplitude targets can be resolved only if they are spaced greater than about 2.5 pulse widths.[60] One reason for the poorer resolution is that the range cells adjacent to the test cell in Fig. 5.7 are not used as part of the reference cells since the target energy in the test cell spills over to nearby cells and affects the threshold. Another reason for degraded resolution is that in automatic detection systems, a large target can be detected in adjacent range cells, adjacent azimuth antenna beams, and adjacent elevation beams. Automatic detection systems have to merge the many detections of the same target into a single centroided detection. Trunk[61] has shown, using a generalized likelihood approach, that in theory it is possible to resolve two equal nonfluctuating targets with separations varying between 1/4 and 3/4 of a pulse width, depending on the relative phase between the two echo signals. This requires that the shape of the received echo pulse be known. (Trunk's result is a lower bound. It indicates what might be achieved, but does not necessarily apply to a specific CFAR configuration.)

Nonparametric Detectors A common assumption in most CFARs is that the statistics of the clutter or noise in the reference cells are known (usually taken to be Rayleigh), except for a scale factor. In many cases, however, the form of the clutter probability density function is not known. A *nonparametric detector*, also known as a *distribution-free detector*, has been considered as a CFAR when the clutter statistics are not known.[62] Its operation is not described here since it is seldom used. The nonparametric detector has a relatively large CFAR loss and problems with correlated samples. In addition, it is fairly susceptible to target suppression from large targets in the reference cells, its implementation might not be simple, and there is loss of amplitude information.[63]

Clutter Map[64–66] A clutter map divides the radar coverage area into range-azimuth cells on a polar or a rectangular grid. The clutter echo stored in each cell of the map then can be used to establish a threshold for that range and azimuth. It is, therefore, a form of CFAR.

The size of each clutter-map cell is equal to or greater than the radar resolution. At each of the range-azimuth cells of the clutter map, a number proportional to the amplitude of the clutter within the cell is stored in the map memory. Since clutter can change with time, the value of the clutter in each cell is updated periodically by averaging over a large number of scans (for example, 5 to 10 scans). The larger the number of scans the more accurate will be the estimate of the clutter, the lower the loss, and the less the effect of a target that moves through the cell. On the other hand, the averaging time (determined by the number of scans) should be shorter than the limited dwell time in which moving clutter (rain or chaff) is within the cell. A short averaging time also allows the threshold to recover to its proper state within a few scans after a target has passed through the cell.

A clutter map CFAR has an advantage over the CA CFAR in that it is not affected by nonhomogeneous clutter (edge effects). The response of the clutter map CFAR will be reduced when a target of slow speed remains within the cell long enough to affect the threshold. This effect can be reduced by making the map cell greater than the radar

resolution cell.[65] Increasing the size of the clutter-map cell should not be carried too far, however, since it reduces the interclutter visibility.

The loss of signal-to-noise ratio in the clutter map will depend on the averaging time. The longer the time, the less the loss. In a particular example, Khoury and Hoyle[64] state that the loss is 0.8 dB when the averaging time is approximately 2 minutes.

Another attribute of the clutter map is the elimination of those resolution cells containing slowly moving objects such as birds.[67] Each threshold crossing is checked against a clutter map before initiating an acquisition. It has been said that even with bird densities as low as 0.1 to 0.2 birds/km^2 a radar tracker can be overloaded and waste much of its time on birds.

The clutter map used in the original MTD discussed in Sec 3.6 was not a true CFAR. It could be called a *blanking clutter map* since it passes targets whose amplitude exceeds that of the clutter.[64]

Other Forms of CFAR Forms of CFAR that predate the CA CFAR include the following:

- *Siebert CFAR:* The output of a postdetection integrator (low-pass filter) provides an estimate of the average noise level which is then applied as a feedforward signal to control the threshold level to maintain the false-alarm rate constant.[68,69] This was one of the first attempts to provide a CFAR, and it was employed in the AN/FPS-23 bistatic CW radar installed by the U.S. Air Force on the DEW (Distant Early Warning) line in the middle 1960s for automatic detection of low-flying aircraft.
- *Hard limiter:* An example is the so-called *Dicke fix,* which consists of a broadband IF filter followed by a hard limiter (which is set low enough to limit receiver noise) and a narrowband matched filter.[69] The output is then unaffected by the amplitude of the noise. The Dicke fix is especially effective against impulse-like noise and broadband jamming. It would not normally be used with an MTI radar since, as mentioned in Sec 3.7, hard limiting restricts the improvement factor that can be achieved.
- *Log-FTC:* This is described in Sec. 7.8. It is a CFAR when the noise or clutter has a Rayleigh probability density function.

CFAR Use in Radar CFAR is used in radars to maintain effectiveness when there are too many extraneous crossings of a fixed threshold caused by noise or clutter. Automatic tracking of targets can be seriously degraded if excessive false alarms occur.

CFAR is to a radar as crutches are to a person with a broken foot. The crutches allow the person to be mobile, but they are something the person would rather not need. CFAR may allow a radar to continue operation, but there are limitations in performance that accompany its use. CFAR automatically adjusts the threshold to prevent threshold crossings that tie up and overload the tracking computer. Increasing the threshold to maintain a constant probability of false alarm, however, lowers the probability of detection and results in missed detections of some targets. This loss of targets has to be tolerated when CFAR is used. In addition to missed target detections, CFAR can cause a loss in the signal-to-noise ratio when the statistics of the clutter or noise are not estimated accurately. The leading and trailing edges of some CFARs can produce undesired threshold

crossings (false alarms). Target suppression can occur when one or more targets are within the reference cells. There is poor range resolution compared to a radar without CFAR. Furthermore, those CFAR designs that might be subject to spoofing by hostile electronic countermeasures have to be avoided in military radars. Thus CFAR is a "necessary evil," needed for maintaining operation of automatic detection and tracking systems that would cause excessive false alarms due to noise or clutter.

CFAR would not be required if the radar had good doppler processing to reject clutter, good ECCM (electronic counter-countermeasures) to reject hostile noise jamming, good EMC (electromagnetic compatibility) to reject interference, and a good tracking computer that recognizes (without overloading in the presence of a large number of threshold crossings) desired moving targets and rejects clutter echoes that break through the signal processing.

Doppler-Estimation False-Alarm Control A quite different method of controlling false alarms is to estimate the target amplitude and the radial velocity of the target (from a measurement of the doppler frequency shift).[70] Noise or clutter are discriminated from targets by the variation in the radial velocity and amplitude over successive measurements. Consistency tests are applied to the measurements based on the assumption that clutter and noise will fluctuate in both amplitude and estimated doppler over successive measurements; but moving targets generally will not. Also, multiple measurements at different pulse repetition intervals and/or frequencies can be used to produce an unambiguous velocity estimate from which moving targets can be separated from stationary clutter to aid in the tracking process. No reference cells are required in this method, so that the problems of nonhomogeneous environments that degrade the CA CFAR (edge effects and multiple targets) do not appear.

5.8 THE RADAR OPERATOR

The discussion in this chapter assumed automatic detection without an operator making the detection decision. Modern radars usually make the detection decision automatically. An operator viewing a display can be a good detector of targets, as has been demonstrated in the past.[71] On the other hand, operators do not have the capacity to process large quantities of information as rapidly as do electronic circuits, and they can become fatigued.

It has been demonstrated experimentally that when an operator views a display in which the pulses received from successive sweeps are presented side-by-side without saturating the display and without fading, the integration improvement achieved by an operator is equivalent to what would be expected from classical detection theory.[72,73]

To obtain the benefits of both automatic detection and the capability of an operator to interpret unusual situations, some radar designers prefer to make the raw video information available to the operator along with the automatically processed information.

5.9 SIGNAL MANAGEMENT

This chapter has been concerned with the detection of desired radar signals. To close the chapter and introduce the next one on the extraction of radar information, we briefly list below the various parts of *signal management* that occur throughout the radar system. Signal management includes everything associated with the waveforms and their processing that is required for a radar to do its job of detecting and locating targets and determining something about their nature. Signal management starts with the design of a suitable waveform and its radiation into space, the collection by the receiver of echo signals reflected from targets and other objects, the use of signal processing to extract the desired signal and reject undesired echoes, the use of data processing to extract information about the detected signals, the coordinated control of these processes throughout the radar, and keeping within the resources and constraints that affect signals and their management. Some parts of signal management listed below apply to a conventional pulse radar with an envelope detector; some apply to a radar that extracts moving targets based on their doppler frequency shift; and some to both.

Figure 5.9 indicates the various factors that enter into radar signal management.

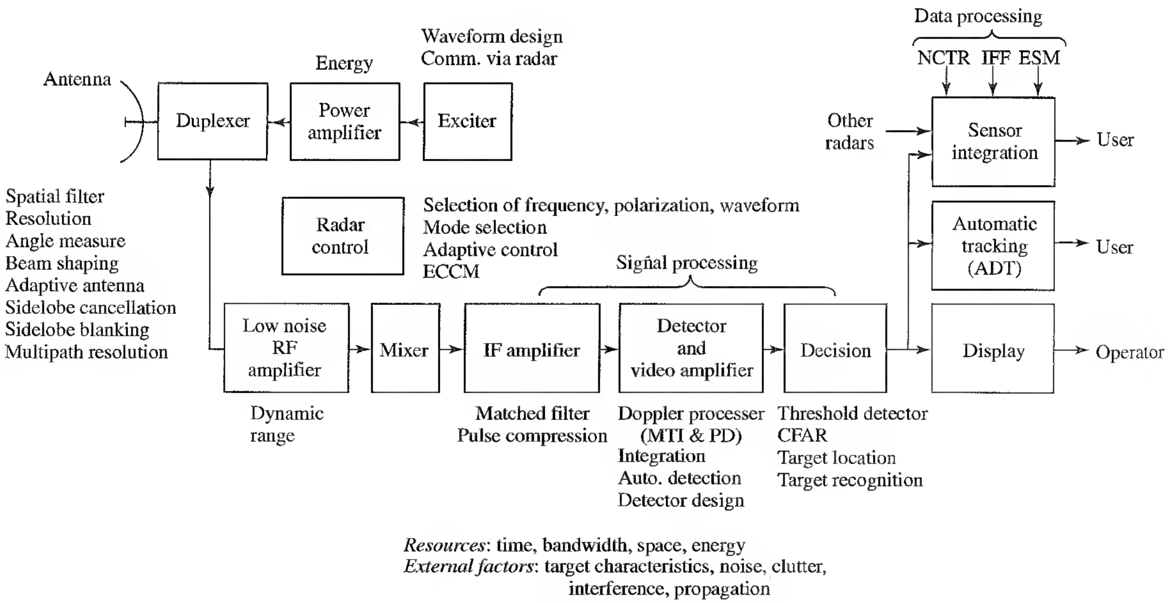


Figure 5.9 The various elements that enter into radar signal management.

Component Parts of Radar Signal Management

Signal Processing This is processing for the purpose of detecting desired echo signals and rejecting noise, interference, and undesired echoes from clutter. It includes the following:

Matched filter: to maximize the signal-to-noise ratio at the output of the radar receiver, and thus maximize detectability of echo signals.

Detector/integrator: the means for processing in a convenient and efficient manner the number of pulses received from a target so as to take full advantage of the total signal energy received from a target.

Clutter reduction: to eliminate or reduce unwanted clutter by one or more methods, of which filtering of moving targets based on the doppler frequency shift is the most important.

CFAR: used to maintain a constant false-alarm rate at the output of the threshold detector when the radar cannot eliminate unwanted echoes.

Electromagnetic compatibility (EMC): the elimination of interference from other radars and other electromagnetic radiations that can enter the radar receiver.

Electronic counter-countermeasures (ECCM): in a military radar, those methods employed to reduce or eliminate the effectiveness of jamming, deception, and other hostile electronic active and passive measures whose purpose is to degrade radar performance. ECCM can be found throughout the radar, not just as part of the signal processing.

Threshold detection: the decision as to whether the output of the radar is a desired signal.

Data Processing These are the processes that take place after the detection of the desired signals for the purpose of acquiring further information about the target.

Target location: in range, angle, and sometimes radial velocity (from the doppler shift). Location information is not generally thought of as either signal processing or data processing. It is usually obtained as part of the detection process (since detection without location is of no value).

Target trajectory: also called *target track,* which is the time history of the target's location. Usually a prediction of the target's future position is included.

Target recognition: the recognition of the type of target being viewed by the radar. It might include the recognition of aircraft from birds, one type of aircraft or ship from another, recognition of various types of weather, and information about the land and sea environment (remote sensing).

Weapon control: in military systems, the use of the radar output for the control and guidance of weapons.

Waveform Design The selection of the waveform depends on what is required of the radar for detection in noise, clutter, interference, and electronic counter-countermeasures,

as well as for the extraction of information from the radar signal. Waveform design will affect the signal and data processing. Multiple waveforms for different purposes can be an important aspect of high-performance radar. The radar signal can also be adapted to communicate with other radars.

Antenna This is not just for radiating and collecting radar signals, but is the means by which angle information is obtained and by which the radar coverage is achieved. The antenna can act as a spatial filter that can affect the spectral properties of wideband signals. It can also provide, in some cases, the angle rate and extract a *spatial* doppler shift similar to the temporal doppler shift.[74] The target's tangential velocity obtained from the spatial doppler shift, along with the radial velocity obtained from the more common temporal doppler shift, can provide the vector velocity of the target.

Automatic Radar Control A radar often employs multiple waveforms and various signal processing options to maximize performance under a variety of environmental conditions. Radar control involves the automatic selection of the proper waveform and signal processing according to the environment and interference (both natural and intentional) encountered by the radar.

Sensor Integration The outputs from other radars covering the same region may be combined to form tracks. Information from the civil aviation Air Traffic Control Radar Beacon Systems (ATCRBS) or the military identification friend or foe (IFF), or other civil or military command and control information can be used to assist in identifying the target. Noncooperative target recognition (NCTR) based on special radar waveforms and processing, as well as signals and information obtained by electronic warfare (EW) methods, such as electronic support measures (ESM), may be used as part of an integrated military target-recognition system.

Resources for Signal Management The radar engineer has available the following resources for pursuing the management of signals and extraction of information.

Energy Sufficiently large transmitted energy is important for detection of weak signals in noise at long range and for obtaining accurate radar measurements.

Bandwidth This is the classical measure of information and is especially important for accurate range measurement and the temporal resolution of targets.

Time Time is necessary for accurate measurement of the doppler frequency. Time also is a means for obtaining increased energy when peak power is a limitation. It is important for achieving multiple functions from a single-beam radar within a required time, and for handling the processing of many echo signals.

Space This applies to the physical aperture area required for an antenna. The larger the antenna aperture the greater the echo energy at the receiver and the more accurate the spatial measurements that can be obtained.

Constraints It is not always possible or practical to obtain the desired energy, bandwidth, time, and spatial extent that might be required. Furthermore, the environment might cause clutter echoes that limit a radar's performance: at high microwave and millimeter wave frequencies atmospheric attenuation can be a nuisance; atmospheric refraction can produce both good and bad effects, as discussed in Chap. 8; and the curvature of the earth limits the range of a radar to targets within the line of sight. Military radars must be able to perform their mission in spite of hostile actions designed to degrade or eliminate their effectiveness. In most applications there are constraints imposed by size, space, weight, and perhaps primary power. Spectrum availability is always a consideration and can seriously limit what the engineer might do. There is also the ever-present constraint imposed by cost.

Engineers always have constraints on what they can do and almost never have everything they need to accomplish the desired task. The essence of successful engineering, however, involves compromise so as to provide in a timely manner a new and useful capability at an acceptable cost.

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PROBLEMS

5.1 (a) Find the matched-filter frequency response function $H(f)$ for a perfectly rectangular (video) pulse of duration τ, and amplitude A. (Assume the pulse extends in time from $-\tau/2$ to $+\tau/2$). (b) Sketch (roughly) its magnitude $|H(f)|$ for positive frequencies. (c) Sketch (roughly) the output of the video matched filter. (This can probably be done much easier by "inspection" than calculation.) All right to take $t_m = 0$.

5.2 (a) Find the matched-filter frequency response function $H(f)$ for a perfectly rectangular pulse of sinewave of duration τ, amplitude A, and frequency f_0. (Assume the pulse extends in time from $-\tau/2$ to $+\tau/2$). (b) Sketch (roughly) its magnitude $|H(f)|$ for positive frequencies. (c) Sketch (roughly) the output of the matched filter. (A rough sketch means it does not need to be precise or "artistic".) (d) Optional—In parts (a) and (b), your expression for $H(f)$ probably contained negative frequencies. What is the meaning of negative frequencies in $H(f)$ and what does one do about them? [Note that the answer to part (d) is not obvious or readily found in textbooks, and might require a little basic thinking about a Fourier transform and what it really is.]

5.3 From problem 5.1a you found that the output of a filter matched for a single *video* rectangular pulse of width τ and amplitude A is a triangular pulse with peak amplitude $A^2\tau$ and whose base has a width 2τ. (a) Sketch and label the output of a filter matched to a train of three equal video rectangular pulses with spacing T between pulses. (This can be done by inspection rather than by calculation.) (b) The more usual way to process a number of pulses, as discussed in Chap. 2, is to pass each pulse in sequence through a filter matched to a *single* pulse and integrate (add) the total number of pulses either coherently or noncoherently. Sketch and label the integrated output of a train of three equal video pulses when processed in this manner.

5.4 Find the ratio of the peak-signal-to-mean-noise power out of a matched filter designed for an RF signal

$$s(t) = Ae^{-at} \sin 2\pi f_0 t$$

where $0 < t < \tau$, and A and a are constants. The input noise is white and of spectral density N_0. You may assume there are many cycles of f_0 within the pulse duration τ, and that $e^{-a\tau}$ is small. (You should use integral tables.)

5.5 The input signal to its matched filter is $s(t) = (A/T)\,(T - t)$, where $0 \le t \le T$. Sketch the following: (a) the input signal, (b) the impulse response of the matched filter, and (c) the output signal from the matched filter. (d) Why is this particular waveform unrealizable?

5.6 What are the units of the constant G_a in the expression for the matched filter frequency response function given by Eq. (5.1)?

5.7 This problem involves finding the matched filter for fixed clutter modeled as nonwhite noise (NWN). The clutter power is assumed to be much larger than receiver noise so that the clutter echo rather than receiver noise determines signal detectability. It is assumed that the clutter is uniformly distributed and stationary so that the power spectral density $|N_i(f)|^2$ of the clutter echo signal can be considered to be the same as the power spectrum of the transmitted radar signal which is reflected from it. (a) Starting with Eq. (5.19) find the frequency response function $H(f)$ of the NWN matched filter for detecting a stationary point target in clutter as given by the above assumptions. (b) If the radar signal $s(t)$ were a perfectly rectangular pulse of width τ, sketch $|H(f)|$ for the NWN matched filter. (c) Why is this clutter matched filter not practical? (d) Optional—If you never heard of a matched filter, what type of radar waveform might you have selected to attempt to detect a stationary point target in uniform distributed clutter much larger than receiver noise? (e) If you answered part (d), how might your solution compare (better or worse) to the NWN matched clutter filter of part (b)?

5.8 This concerns the effectiveness of a nonmatched filter. (a) Find the peak-signal-to-mean-noise ratio (SNR) out of a one-stage low-pass RC network when the input is a rectangular pulse of width τ, amplitude A = 1, and the noise is white with a noise power per unit bandwidth of N_0. The normalized frequency response function is

$$\text{frequency response function of low-pass RC network} = H(f) = \frac{1}{1 + jf/B_v}$$

where B_v = bandwidth of the low-pass filter. Note that the maximum SNR occurs at a time equal to the pulse width τ. (b) Find the peak-signal-to-mean-noise power out of a filter that is perfectly matched to the rectangular pulse. (c) What is the loss in SNR (in dB) introduced by the nonmatched filter of (a) compared to the matched filter of (b)? (d) If the efficiency of the nonmatched filter relative to that of a matched filter is defined as

$$\rho_f = \frac{|s_0(t)|^2_{\max}/N_{\text{out}}}{2E/N_0}$$

what is the value of $B_v\tau$ that maximizes the efficiency? [Note that a low-pass RC video network produces results for the above that are equivalent to what would be obtained with a single tuned RLC resonant network as might be used in the IF, assuming $B_v = B_{IF}/2$. Thus your answer to part (d) also applies to a single tuned RLC resonant network that could be in the IF portion of the receiver. A single-tuned circuit, however, is seldom found in radar receivers, so the answers you obtain in this problem might not be typical for radar.]

5.9 (a) Draw the block diagram of a correlation receiver. (b) Explain why the correlation receiver can be considered equivalent to the matched filter receiver in detection performance.

(c) Under what conditions, if any, might one choose to implement a correlation receiver rather than a matched filter receiver?

5.10 Sketch the matched-filter frequency response function when the waveform is just one RF cycle of sinewave in duration. You may start with the answer you found for problem 5.2(a). (A single cycle sinewave is an *ultrawideband* waveform.)

5.11 The matched filter of Eq. (5.1) assumed that the shape of the radar echo was the same as the shape of the transmitted radar signal. When a target is observed by a high-resolution radar (one with a range-resolution cell size much smaller than the target's radial extent), the target echo is not the same as that which was transmitted. It will consist of the superposition of echoes from the individual scattering centers of the target. (An example is a large ship 500 feet in length being observed head-on by a civil marine radar using a pulse width of 80 ns.) Discuss what has to be considered about the "matched filter" when attempting to detect a target that is much longer in radial size than the range-resolution cell so that the target echo is resolved into multiple scatterers. (Note that this question does not have a simple, unique answer.)

5.12 Show that the impulse response of a matched filter $[h(t) = G_a s(t_m - t)]$ is the inverse Fourier transform of its frequency response function $H(f) = G_a S^*(f) \exp(-j2\pi f t_m)$.

5.13 Why is a CFAR needed in some radars? What are the disadvantages of using CFAR?

5.14 What does one have to do in a radar system to avoid the use of a conventional CFAR?

5.15 In the VHF frequency region (30 to 300 MHz), the external noise at the receive-antenna terminals is generally higher than receiver internal noise. If one were to design an ultrawideband radar at VHF, qualitatively describe how the matched filter of Eq. (5.1), based on white noise, would have to be modified to allow for the large external noise levels that vary with frequency? (You might want to review Sec. 8.8 or other related sources on external noise.)

5.17 Show that the optimum detector law based on the criterion of the likelihood-ratio receiver is $y = \ln I_0(av)$, where y is the receiver output, a is the amplitude of the received sinewave signal normalized (divided) by the rms noise voltage, v is the amplitude of the IF voltage envelope normalized by the rms noise voltage, and $I_0(x)$ is the modified Bessel function of zero order. [The following outlines how you might work through the derivation. Start with the likelihood ratio of Eq. (5.21). Assume there are N independent pulses with normalized envelope-amplitudes $v_1, v_2, \ldots, v_N$ available from the radar receiver. The probability density function for the ith noise pulse $p_n(v_i)$ is found from Eq. (2.21), where v_i is the ratio $R/\psi_0^{1/2}$, R is the envelope amplitude of the ith output and $\psi_0^{1/2}$ is the rms noise level. The probability density function for the ith signal-plus-noise pulse $p_s(v_i)$ is found from Eq. (2.27), with a = ratio of the sinewave signal amplitude to rms noise. The likelihood ratio of the N pulses is

$$L_r(v) = \frac{\prod_{i=1}^{N} p_s(v_i)}{\prod_{i=1}^{N} p_n(v_i)} \geq K$$

where K is the receiver threshold level. After making the substitutions one should take the log of both sides so that the product becomes a more convenient sum. At this point, examination of the likelihood ratio will show how the signal should be processed and indicate the nature of the detector law.] How does this "optimum detector" relate to more conventional detectors?

5.18 What are the advantages and limitations of a binary integrator?

5.19 How does the performance of a radar operator making detection decisions by viewing the raw (unprocessed) video output of a radar display compare to the performance of an automatic (electronic) detector?